S

Subspace Methods

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Synonyms

Multiple similarity method

Related Concepts

- ▶ Dimensionality Reduction
- ▶ Principal Component Analysis (PCA)

Definition

Subspace analysis in computer vision is a generic name to describe a general framework for comparison and classification of subspaces. A typical approach in subspace analysis is the subspace method (SM) that classifies an input pattern vector into several classes based on the minimum distance or angle between the input pattern vector and each class subspace, where a class subspace corresponds to the distribution of pattern vectors of the class in high-dimensional vector space.

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Background

Comparison and classification of subspaces have been one of the central problems in computer vision, where an image set of an object to be classified is compactly represented by a subspace in high-dimensional vector space.

The subspace method is one of the most effective classification method in subspace analysis, which was developed by two Japanese researchers, Watanabe and Iijima around 1970, independently [1, 2]. Watanabe and Iijima named their methods the CLAFIC [3] and the multiple similarity method [4], respectively. The concept of the subspace method is derived from the observation that patterns belonging to a class form a compact cluster in high-dimensional vector space, where, for example, a $w \times h$ pixels image pattern is usually represented as a vector in $w \times h$-dimensional vector space. The compact cluster can be represented by a subspace, which is generated by using Karhunen-Loève (KL) expansion, also known as the principal component analysis (PCA). Note that a subspace is generated for each class, unlike the Eigenface Method [5] in which only one subspace (called eigenspace) is generated.

The SM has been known as one of the most useful methods in pattern recognition field since its algorithm is very simple and it can handle classification of multiple classes. However, its classification performance was not sufficient for many applications in practice, because class subspaces are generated independently of each other [1]. There is no reason to assume a priori that each class subspace is the optimal linear class subspace in terms of classification performance.

To deal with this problem, the SM has been extended. Two typical extensions are the orthogonal subspace method and the learning subspace methods. The orthogonal subspace method [6] executes the SM to a set of class subspaces that are orthogonalized based on the framework proposed by [7] in learning phase. The orthogonalization is known as a useful operation to boost the performance of angle-based method, such as SM, since class subspaces are usually close to each other in many classification problems.

The learning subspace methods [1, 8, 9] execute the SM to a set of class subspaces, the boundaries between which are adjusted to suppress classification errors for the learning pattern vectors. This adjustment is performed based on the following procedure. First, a learning vector $\mathbf{x}$ is classified by using the SM. Then, if $\mathbf{x}$ is wrongly classified into an incorrect class subspace L_r, which is not corresponding to the class of $\mathbf{x}$, subspace L_r is slightly rotated into the direction away from $\mathbf{x}$, and in contrast the correct class subspace L_c of $\mathbf{x}$ is slightly rotated to the direction close to $\mathbf{x}$. This adjustment is repeated several times for a set of learning vectors until a minimum classification error is achieved.

Moreover, to deal with the nonlinear distribution of pattern vectors, the SM had also been extended to the kernel nonlinear SM [10, 11] by introducing a nonlinear transformation defined by kernel functions.

These extensions aim mainly to improve the classification ability. In addition to such extensions, the generalization of the SM to classification of sets of patterns is also important for many computer vision problems. In order to handle a set of multiple pattern vectors as an input, the SM has been extended to the mutual subspace method (MSM) [12]. The MSM classifies a set of input pattern vectors into several classes based on multiple canonical (principal) angles [13, 14] between the input subspace and class subspaces, where the input subspace is generated from a set of input patterns as class subspaces. The concept of the MSM is closely related to that of the canonical correlation analysis (CCA) [13]. Actually, the cosine of the i-th smallest canonical angle corresponds to the i-th largest canonical correlation.

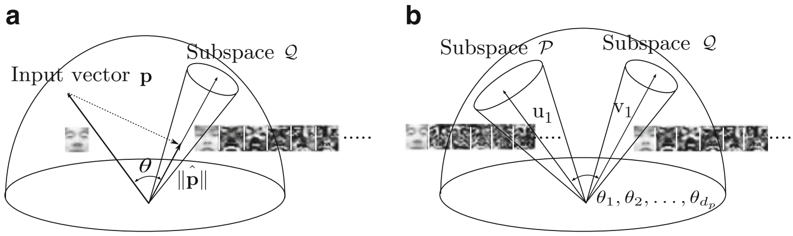
The MSM has achieved high performance in recognition of complicated 3D object such as face, using a set of images from image sequence or multi-view images. This success can be mainly explained by the fact that the MSM implicitly utilizes 3D shape information of objects in classification. This is because the similarity between two distributions of various view images of objects reflects the 3D shape similarity between the two objects. To boost the performance of the MSM, it has been further extended to the constrained mutual subspace method (CMSM) [15, 16] and the whitening (or orthogonal) mutual subspace method (WMSM) [17], where the relationship among class subspaces is modified to approach orthogonalization in the learning phase. In CMSM, the orthogonalization is performed by projecting the class subspaces onto a generalized difference subspace [16], which represents difference components among the class subspaces. In WMSM, it is performed by whitening all the class subspaces. These extensions have boosted the classification ability of the MSM. The MSM and its extensions have been further extended to kernel nonlinear methods [18–21] by kernel trick.

Theory

Subspace Method

Assume an input vector $\mathbf{p}$ and k class subspaces in f-dimensional vector space. The similarity S of the pattern vector $\mathbf{p}$ to the i-th class is defined based on either of the length of the projected input vector $\hat{\mathbf{p}}$ on the i-th reference subspace [3] or the minimum angle [4] between the input vector $\mathbf{p}$ and the i-th class subspace as shown in Fig. 1a. The length of an input vector $\mathbf{p}$ is often normalized to 1.0. In this case, these two criteria coincide. In the following explanation, therefore, the angle-based similarity S defined by the following equation will be used:

Subspace Methods, Fig. 1 Conceptual illustrations of SM and MSM. (**a**) Subspace method (SM). (**b**) Mutual subspace method (MSM)



$$S = \cos^2\theta = \sum_{i=1}^{d_q} \frac{(\mathbf{p} \cdot \boldsymbol{\phi}_i)^2}{||\mathbf{p}||^2}, \qquad (1)$$

where d_q is the dimension of the class subspace and $\boldsymbol{\phi}_i$ is the i-th f-dimensional orthogonal normal basis vector of the class subspace, which are obtained from applying the principal component analysis (PCA) to a set of patterns of the class. Concretely, these orthonormal basis vectors can be obtained as the eigenvectors of the correlation matrix $\sum_{i=1}^{l} \mathbf{x}_i \mathbf{x}_i^\top$ calculated from the l learning patterns $\{\mathbf{x}\}$ of the class.

Process Flow of the SM

The whole process of the SM consists of a learning phase and a recognition phase.

In the Learning Phase All k class d_q-dimensional subspaces are generated from a set of pattern vectors of each class by using PCA.

In the Recognition Phase The similarities S of an input vector $\mathbf{p}$ to all the k class subspaces are calculated by using Eq. (1). Then, the input vector is classified into the class of the class subspace with highest similarity. If the highest similarity is lower than a threshold value fixed in advance, the input vector is classified into a reject class.

Mutual Subspace Method

Assume an input subspace and class subspaces in f-dimensional vector space. The similarity of the input subspace to the i-th class subspace is defined based on a minimum canonical angle θ_1 [13, 14] between the input subspace and the class subspace, as shown in Fig. 1b.

Given a d_p-dimensional subspace $\mathcal{P}$ and a d_q-dimensional subspace $\mathcal{Q}$ (for convenience, $d_p \leq d_q$) in the f-dimensional vector space, the canonical angles $\{0 \leq \theta_1, \ldots, \theta_{d_p} \leq \frac{\pi}{2}\}$ between $\mathcal{P}$ and $\mathcal{Q}$ are uniquely defined as [14]

$$\cos^2\theta_i = \max_{\substack{\mathbf{u}_i \perp \mathbf{u}_j (i \neq j,\ i, j = 1 \sim d_p) \\ \mathbf{v}_i \perp \mathbf{v}_j (i \neq j,\ i, j = 1 \sim d_p)}} \frac{(\mathbf{u}_i \cdot \mathbf{v}_i)^2}{||\mathbf{u}_i||^2 ||\mathbf{v}_i||^2}, \qquad (2)$$

where $\mathbf{u}_i \in \mathcal{P}$, $\mathbf{v}_i \in \mathcal{Q}$, $\|\mathbf{u}_i\| \neq 0$, $\|\mathbf{v}_i\| \neq 0$, $(\cdot)$ and $\| \ \|$ represent an inner product and a norm, respectively.

Let $\boldsymbol{\Phi}_i$ and $\boldsymbol{\Psi}_i$ denote the i-th f-dimensional orthonormal basis vectors of the subspaces $\mathcal{P}$ and $\mathcal{Q}$, respectively. A practical method of finding the canonical angles is by computing the matrix $\mathbf{X}$=$\mathbf{A}^\top\mathbf{B}$, where $\mathbf{A} = [\boldsymbol{\Phi}_1, \ldots, \boldsymbol{\Phi}_{d_p}]$ and $\mathbf{B} = [\boldsymbol{\Psi}_1, \ldots, \boldsymbol{\Psi}_{d_q}]$. Let $\{\kappa_1, \ldots, \kappa_{d_p}\}$ $(\kappa_1 \geq, \ldots, \geq \kappa_{d_p})$ be the singular values of the matrix $\mathbf{X}$. The cosines of canonical angles $\{\theta_1, \ldots, \theta_{d_p}\}$ can be obtained as $\{\kappa_1, \ldots, \kappa_{d_p}\}$. The original MSM uses only a minimum canonical angle θ_1 to define the similarity. However, since the remaining canonical angles also have information for classification, the value, $\tilde{S} = \frac{1}{t}\sum_{i=1}^{t} \cos^2\theta_i$, defined from the smallest t canonical angles is often used as the similarity in many computer vision problems. The value $\tilde{S}$ reflects the structural similarity between two subspaces. The whole process of the MSM is the same as that of the SM except that an input vector is replaced by an input subspace.

Constrained Mutual Subspace Method

The essence of constrained mutual subspace method (CMSM) [15, 16] is to conduct MSM on a generalized difference subspace (GDS), where the GDS is generated from the sum of the orthogonal projection matrices of all the class subspaces [16]. This can be performed by applying MSM to a set of an input subspace and class subspaces, which are projected onto the GDS. For the projection, there are two ways that give equivalent results. One is to project the basis vectors of each class subspace onto GDS and then normalize the projected basis, which is further followed by Gram-Schmidt orthogonalization after the orthonormality is lost by the projection. The alternative is to first project the images for each class subspace and then generate a class subspace from the projected images.

Application

The subspace methods and their extensions have been applied to various problems [1, 10, 11] of computer vision due to their high general versatility and low computational cost. In particular, the extended SMs have produced remarkable results in optical character recognition (OCR), such as handwriting Chinese character recognition [2, 4], in Japanese industry.

The mutual subspace method has also been demonstrated to be extremely effective for 3D object recognition. In particular, the MSM has been known to be suitable for face recognition [15, 17, 22] because the subspace (called "illumination subspace"), which includes any face image patterns under all possible illumination conditions, can be generated from face images under more than three different illumination conditions [23]. The nonlinear extensions of the MSM, CMSM, and WMSM have been shown to be further effective for 3D object recognition using image sequences and multi-view images [18–21, 24, 25]. These methods work well together with CNN features, which are extracted through a pre-trained Convolution neural network [26].

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